Minimizing a DFA

Lecture 9
Section 2.4

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Mon, Sep 12, 2016

Outline

1. Indistinguishable States
2. The Algorithm
3. Minimization Examples
4. Assignment

Outline

Indistinguishable States

Definition (Indistinguishable states)

Two states p and q in a DFA are indistinguishable if, for all $w \in \Sigma^*$,

$$\delta^*(p, w) \in F \Leftrightarrow \delta^*(q, w) \in F.$$

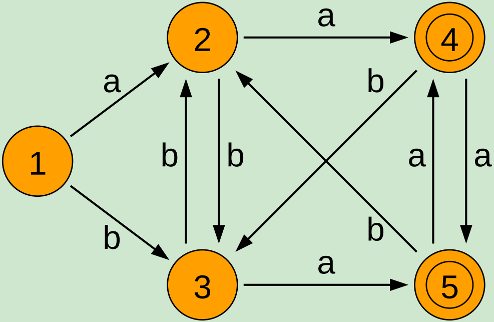
- That is, the decision of whether to accept or reject any input will be the same regardless of which of the two states we are currently in.
- To minimize a DFA, we will identify states that are indistinguishable.
- When two states are indistinguishable, one of them may be eliminated.

Indistinguishable States

- Indistinguishableness is an equivalence relation.
  - Every state is indistinguishable from itself.
  - If p is indistinguishable from q, then q is indistinguishable from p.
  - If p is indistinguishable from q, and q is indistinguishable from r, then p is indistinguishable from r.

Example

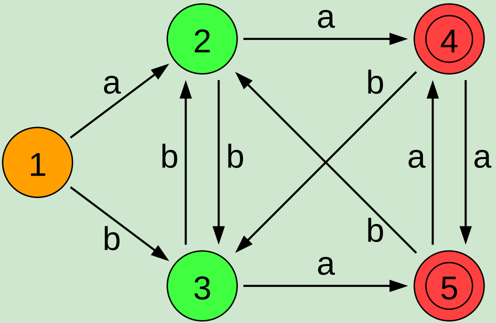
Example (Indistinguishable states)



Clearly, states 2 and 3 are indistinguishable and states 4 and 5 are indistinguishable.

Example

Example (Indistinguishable states)



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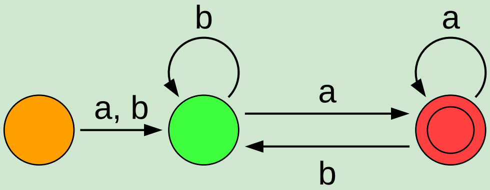
Example

Example (Indistinguishable states)



Clearly, states 2 and 3 are indistinguishable and states 4 and 5 are indistinguishable.

Outline

1. Indistinguishable States
2. The Algorithm
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Determining Indistinguishable States

- To determine which states are indistinguishable,
  - Add a trap state, if necessary, to make the DFA fully defined.
  - Begin with two equivalence classes: F, $Q - F$.
  - This divides Q into two equivalence classes whose members are indistinguishable by "reading λ."

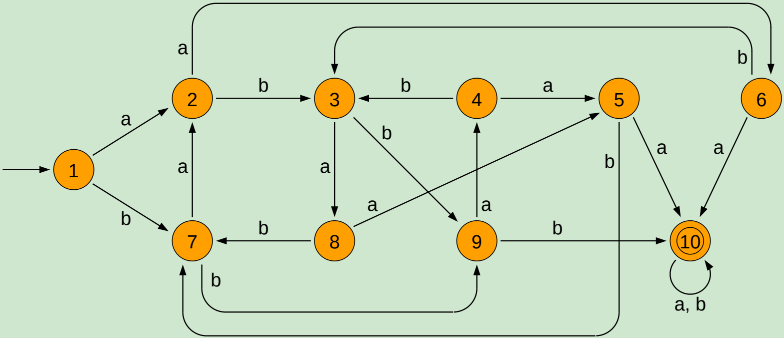
Determining Indistinguishable States

- Within each class, apply a single transition for each symbol in Σ to see which states are distinguishable.
- This divides Q into equivalence classes whose members are indistinguishable by reading a single input symbol.
- Continue in this manner until the next input symbol, no matter what is it, does not distinguish any states.

Outline

Example

Example (Minimizing a DFA)



Minimize this DFA

Example

Example (Minimizing a DFA)

- The initial equivalence classes are

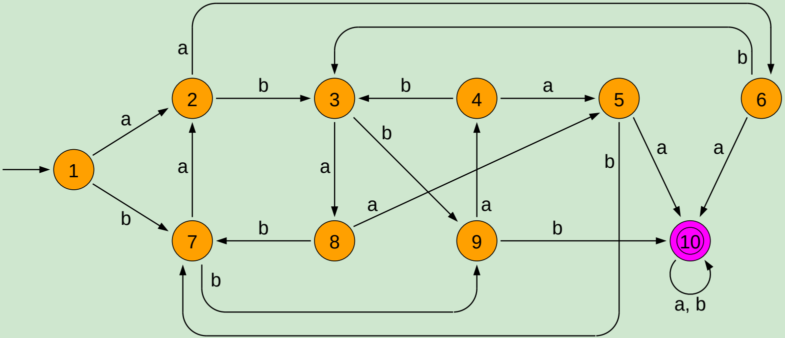
$$F = \{10\}$$

and

$$Q - F = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

Example

Example (Minimizing a DFA)



$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \{10\}$$

Example

Example (Minimizing a DFA)

- Summarize the transitions in the following tables.

A

	1	2	3	4	5	6	7	8	9
a	2	6	8	5	10	10	2	5	4
b	7	3	9	3	7	3	9	7	10

B

	10
a	10
b	10

- Identify each entry with one of the initial equivalence classes

A

	1	2	3	4	5	6	7	8	9
a	A	A	A	A	B	B	A	A	A
b	A	A	A	A	A	A	A	A	B

B

	10
a	B
b	B

Example

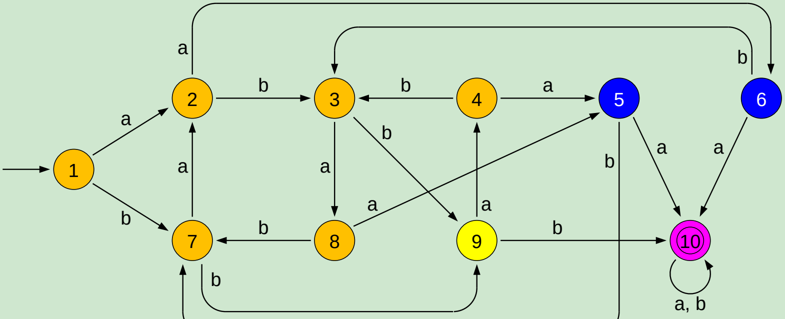
Example (Minimizing a DFA)

- There are three patterns within $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are *AA*, *BA*, and *AB*.
- These patterns subdivide the initial classes into the equivalence subclasses

$$\{1, 2, 3, 4, 7, 8\}, \{5, 6\}, \{9\}, \{10\}.$$

Example

Example (Minimizing a DFA)



$$\{1, 2, 3, 4, 7, 8\}, \{5, 6\}, \{9\}, \{10\}$$

Example

Example (Minimizing a DFA)

A

	1	2	3	4	7	8
a	2	6	8	5	2	5
b	7	3	9	3	9	7

B

	5	6
a	10	10
b	7	3

C

	9
a	4
b	10

D

	10
a	10
b	10

Identify each entry with an equivalence subclass.

A

	1	2	3	4	7	8
a	A	B	A	B	A	B
b	A	A	C	A	C	A

B

	5	6
a	D	D
b	A	A

C

	9
a	A
b	D

D

	10
a	D
b	D

Example

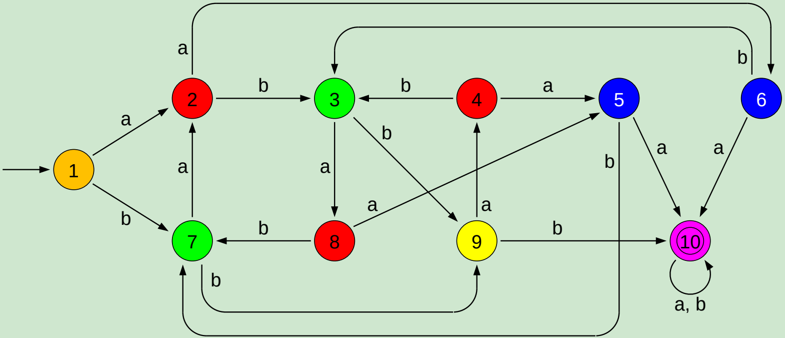
Example (Minimizing a DFA)

- There are 3 different patterns within $\{1, 2, 3, 4, 7, 8\}$: *AA*, *BA*, and *AC*.
- These patterns subdivide this equivalence class into three equivalence subclasses, yielding

$$\{1\}, \{2, 4, 8\}, \{3, 7\}, \{5, 6\}, \{9\}, \{10\}.$$

Example

Example (Minimizing a DFA)



$$\{1\}, \{2, 4, 8\}, \{3, 7\}, \{5, 6\}, \{9\}, \{10\}$$

Example

Example (Minimizing a DFA)

A

	1
a	2
b	7

B

	2	4	8
a	6	5	5
b	3	3	7

C

	3	7
a	8	2
b	9	9

D

	5	6
a	10	10
b	7	3

E

	9
a	4
b	10

F

	10
a	10
b	10

A

	1
a	B
b	C

B

	2	4	8
a	D	D	D
b	C	C	C

C

	3	7
a	B	B
b	E	E

D

	5	6
a	F	F
b	C	C

E

	9
a	B
b	F

F

	10
a	F
b	F

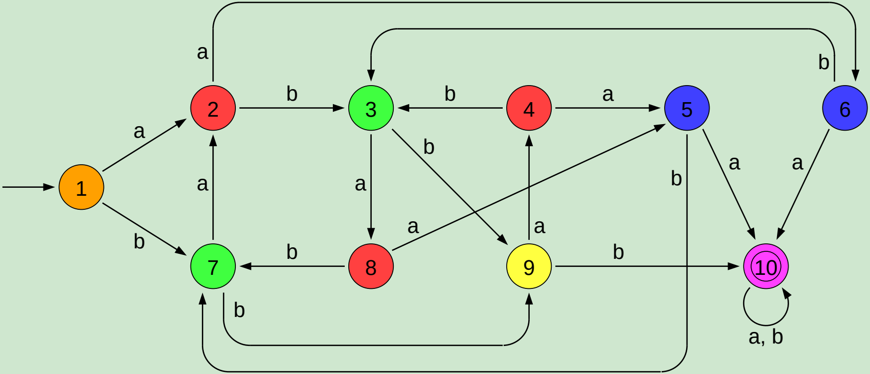
Example

Example (Minimizing a DFA)

- Identify each entry with an equivalence subclass.
- The patterns are the same within each class.
- There is no further subdividing.
- Therefore, the final equivalence classes are

$$\{1\}, \{2, 4, 8\}, \{3, 7\}, \{5, 6\}, \{9\}, \{10\}.$$

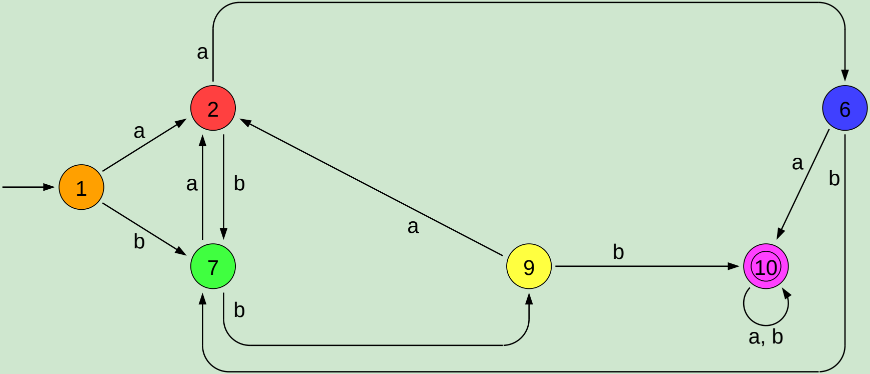
Example (Minimizing a DFA)



The equivalence classes of indistinguishable states

Example

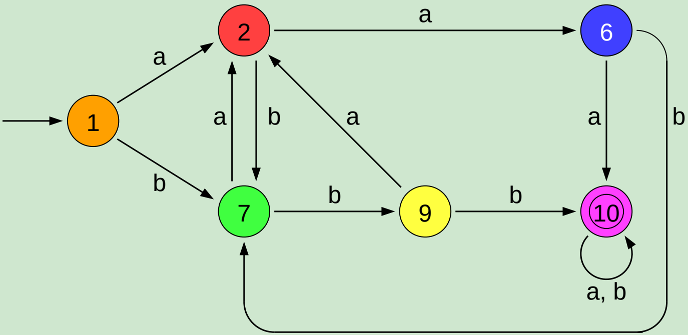
Example (Minimizing a DFA)



The minimized diagram

Example

Example (Minimizing a DFA)

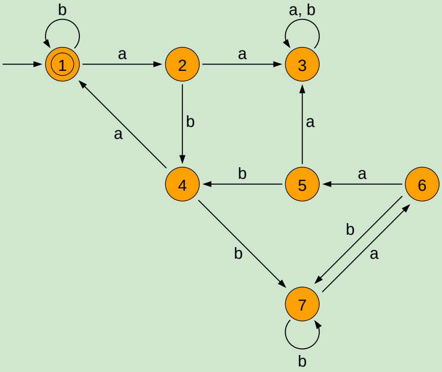


The minimized diagram

Example

Example (Minimizing a DFA)

- Minimize the following DFA.



Example

Minimizing a DFA

- Let $\Sigma = \{\mathbf{a}, \mathbf{b}\}$ and

$$L_1 = \{w \mid w \text{ starts with } \mathbf{a} \text{ and has an even number of symbols}\}$$
$$L_2 = \{w \mid w \text{ starts with } \mathbf{b} \text{ and has an odd number of symbols}\}$$

- Construct a minimal DFA for $(L_1 \cup L_2)^*$.

Outline

Assignment

- Construct an NFA for the concatenation $L_1 L_2$ of the following languages over the alphabet $\{\mathbf{a}, \mathbf{b}\}$ and then minimize it.

$$L_1 = \{w \mid \text{the length of } w \text{ is at most } 1\}$$

$$L_2 = \{w \mid \text{every odd position of } w \text{ is } \mathbf{b}\}.$$